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MATHEMATICAL IDEAS AND INDIGENOUS LANGUAGES

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Abstract

Recent interest in how anthropology and linguistics relates to mathematics has led to recognition that mathematical thinking is a function of language in ways not previously recognised. Ethnomathematics, cognitive linguistics, and anthropology are all pointing to a way of understanding mathematical ideas based on human experience and cultural activities. Formal mathematics can be seen to have developed from metaphors deeply embedded in our languages.

This raises the question of relativity in mathematics. Do different languages embody different types of mathematics? This chapter examines some emerging evidence in the grammar and syntax of indigenous languages, i.e. languages structurally very different from the Indo-European linguistic tradition. The educational consequences of the possibility of different mathematical thinking is briefly discussed.

Introduction

The rise of interest in several quite diverse fields has led to a recent recognition that mathematical thinking is a function of language in ways not previously recognised. Concepts as basic as numbers or geometric objects seem to be conceived in ways which are different from those of conventional mathematics.

The cultural renaissance of indigenous peoples has resulted in a call for schooling in indigenous languages, which, in turn, has resulted in a broadening of the ways in which it is possible to talk about mathematics. A case in point are the Maori of New Zealand. A political renaissance and push for self-determination in the mid-1970s had education as a major initiative, which led to immersion pre-schools, then primary education, and, by the 1990s, there was a demand for Maori language secondary education in all subjects. Interestingly, mathematics was one of the first subject areas to develop a specific register and curriculum in Maori (Barton, Fairhall & Trinick, 1998).

There have been continuing linguistic and anthropological investigations into languages and the 'world-views' they represent. Part of this has been renewed recognition of the work of Benjamin Whorf (Lee, 1998; Whorf, 1956) and the principle of linguistic relativity. This principle states that speakers of languages which are different structurally and grammatically are led to different ways of construing the world. Whorf, and his supervisor Sapir, used evidence from studies of Hopi and English to show how these languages resulted in different interpretations of events, although they believed that all humans had the same basic cognitive processes. Recent extensions of this work can be found in Lucy (1992a & b). Another strand is the work of Lakoff and others in the area of language and cognition. Lakoff (1987) produces a convincing argument against the idea that thoughts correspond to objects in the real world, and for the deep influence of metaphorical thinking in all aspects of human cognition, including mathematics and logic. More recent work (Lakoff & Núñez, 1997) details the way our environment acts to embed cognitive structure.

Within the field of mathematics there has been a long history of attempts to view mathematics as a cultural activity (Struik, 1942; Wilder, 1981; Mac Lane, 1981; Thom, 1992); but these have received renewed impetus with the emergence of ethnomathematics (D'Ambrosio, 1985, 1990; Ascher, 1991; Gerdes, 1986, 1994). The idea that mathematics manifests itself differently in different social or cultural contexts has been embraced by an educational world looking for answers to differences in mathematical achievement. Another educational theme has been a developing literature on bilingual education, including the possibility of cognitive advantage for speakers of more than one language.

It is no surprise, therefore, that questions are being asked about the way in which mathematical ideas are conceived, and are exploring the conventional assumptions about mathematical objects and operations. If mathematics is more relative than has been assumed, where is this relativity? Is there a different mathematics? Why does mathematics have an aura of universality? Why does mathematics seem to correspond with the real world? The thesis of this chapter is that the answers to such questions lie well below the level of usual mathematical activity, they lie within the language used in mathematical talk, and what is more, they lie embedded within the very grammar of that language.

What are we looking for?

Fifty years ago the linguist Benjamin Whorf (1956, p. 245) suggested that different mathematical systems might be sought and would more likely be found in languages fundamentally different from our own, those that he referred to collectively as Standard Average European. Such languages might be geographically remote, spoken by indigenous peoples, that is, linguistic systems that have evolved separately and that have a relatively recent history of interaction with languages such as English, French, or Spanish. The Asian languages, such as Mandarin and Japanese, are different again because their symbols are pictorial rather than phonological. The different ways this might affect mathematical thinking are not part of this paper.

What does "different mathematical systems might be sought in other languages" mean? We are not looking for anything resembling mathematics as it appears in a school textbook. Our view must be widened to embrace mathematical thinking generally. We are looking for what we might call 'QRS systems', that is systems by which we make meaning of quantity, relationships or space. Even this is to presume too much. A language does not contain a system like a measurement system of metres, grams and litres. We are looking for something more fundamental than that.

Lakoff, Núñez, Johnson and others, in a series of works over the last twenty years (Lakoff & Johnson, 1980; Lakoff, 1987; Lakoff & Núñez, 1997; and books in preparation) have presented compelling linguistic evidence for the deep use of metaphor and metonymy in the way humans structure their concepts, including the concepts of mathematics. Their work begins by examining the way we classify concepts and then discusses examples of concepts in linguistics and mathematics. It shows clearly that mathematical concepts cannot be attached to ideal mathematical objects which exist independently and objectively in some world, rather the concepts are developed, through language, from human experiences. In addition, the shape of that development itself derives from fundamental experiences. Experience is embodied in the form of metaphors within the language we use. Some of the fundamental ones in English (Lakoff & Núñez claim that they are universal) include the 'container' metaphor (the idea for sets in mathematics), the 'things in piles' and 'points on line' metaphors (the ideas for numbers), the 'arrow' metaphor (the idea for functions), and the 'turning around' metaphor (used in geometry).

At present there is a lack of theoretical tools and approaches that would cope adequately with the idea that there are no language universals at all. Thus, it is difficult to find ways to even talk about, for example, 'another mathematics' which make sense. Such a concept requires that the development of QRS systems, and other cognitive structures, start very early. The idea that the way in which very young children 'see' their world –precognitive perceptual activity– may be language-based is only recently being discussed (Levinson, 1996).

A leading algebraist, Saunders Mac Lane (1981, p. 465) once described mathematics in these terms: "Mathematics starts from a variety of human activities, disentangles from them a number of notions which are generic and not arbitrary, then formalises these notions... . Thus ... mathematics studies formal structures by deductive methods..."

He goes on to say that it would be possible to construct mathematics using, say, the notion of arrows (something linked directionally to something else) rather than containers (sets), and that if this was done then mathematics would look quite different. We are suggesting that Mac Lane's 'notions' are embedded grammatically and metaphorically in our language of human activities, and, furthermore, that the deductive methods used in mathematics are also prescribed by the metaphors and grammar of our communication. What we are interested in is whether other languages have different metaphors –or, possibly, a greater propensity for one metaphor over another– embedded in their grammar of quantity, relationship or space.

There are various levels on which we might conduct our search. At the most superficial level we can examine the words which make up the language. Then we might look more deeply at the way in which the words are used, that is the syntax and morphology of the language. Finally, we might look at a still deeper level at the ethos of the language, at the metaphors it uses and propensities it encourages, in other words, its repertoire of image schemata (Frank & Susperregi, 2001).

Focus on Vocabulary, Syntax & Grammar

Most work on mathematical ideas in language has focussed on the different number words used around the world, examining them for the implied base used in counting and for relationships between languages. (Menniger, 1969; Lean, 1995). More recent work has had a semiotic focus, examining mathematical ideas and their symbols as communicative acts (e.g. Rotman, 1987). A feature of all of these works is an implied universality in the way numbers, shapes, or operations are thought about and used. Even Crump's *Anthropology of Number* (1989) makes assumptions about what 'number' might be, although it does focus on the role of number in different societies.

Such assumptions are challenged by a closer examination of the way vocabulary is used. Even at a surface level, attention to vocabulary can alert us to different conceptual systems. As part of other work one of the authors asked a Maori weaver about some basket-weaving patterns, and was surprised to discover that several patterns which appeared to bear no relationship to each other were given the same name. To a conventional mathematician these patterns have different symmetry. To the weaver's eyes they are the same because they require the same initial set-up of black and white strands in order to create each pattern. What might be called 'strand symmetry' is so important that it is reflected in the naming system. This classification of pattern cannot be subsumed by the usual classification by line and rotational symmetries. It may be context-specific, but the example at least shows that the usual forms of symmetry are not universally applicable.

As another example Lipka (1994) notes a pattern system based on what mathematicians might call polar coordinates in the basket-making of the Yup'ik in Alaska. It would be interesting to know whether this was confirmed by the presence and manipulation of related conceptual categories from their lexical repertoire. Indeed, such an analysis might be the best way discover to what extent this pattern is generalized by the Yup'ik themselves, e.g. whether there is evidence that they tend to organize other aspects of space in terms of a polar-like symmetry.

Another source for finding QRS systems is amongst those words which do not have equivalents when translated into English. These concepts have referential domains which only partially overlap, or which are non-commensurate, with those of English. In Fiji there are several words which are names for cultural practices which have to do with transferring goods. These practices are not trading in any generally accepted sense of that word, but represent specific practices which have no equivalent. For example, *solevu* is a public, ceremonial exchange of goods between groups; *kerekere* is a form of gifting in response to a request (Bakalevu, 1998). Both of these concepts involve a quantification, but not an accounting as it is understood in European trade terms. For example, it is important that the size of the presentation is known, recognised, and returned in excess; and that a public sharing takes place. Neither of these practices are formalised mathematics, however they are systems which deal with quantity and cannot be adequately represented by the measuring and numerical operations taught as part of a formal mathematics lesson.

There is a suggestion that the dominant position held by the world-view of European languages has affected the way vocabulary items map meaning. In Maori *tonga* means *south*. It has its equivalent in Hawaiian: *kona* meaning *leeward*. In Hawai'i this happens to be a south-westerly direction. It seems likely that this word at some point got fixed to a north-south grid reference, instead of its wind-direction referent (which would be more practical in a Pacific Island context with prevailing winds and local sea travel). The Maori word *muri* means north, and also means *behind*, (being associated with *mua* meaning *in front*). Trinick (1999) suggests that, as the initial migration to New Zealand came from the north-east, so *behind* would be associated with that direction (it is also used to refer to the stern of a canoe). Again, the referent was, at some later time, fixed onto a north-south grid referent, as the orienting direction of 'home' lost its relevance for local travel. Did this re-fixing of the referent occur at the time of European contact –contact with sailors and others who would only think about directions in terms of compass points?

It is likely that specialised navigational vocabulary will hold the key to identifying more systematic differences in the way location is represented. Pacific navigation seems to have been conceived as 'pathways' rather than 'position'; thus, navigating from one place to another is understood as a journey and described by what might be seen or experienced along the way, and how to tell whether one is on the correct track; rather than as a series of positions at any particular time. Sea travel for Pacific navigators was more like a car journey for a modern traveler, than like a chart-plotted sea journey for a yachtsman.

Harris (1991) also uses linguistic evidence to suggest that Australian Aborigines use a north/south/east/west location system even in very local situations such as describing where in a room a piece of furniture is located. The implication is that Aboriginal children are disadvantaged in schools because the curriculum assumes easy familiarity with right/left/front/back orientation systems in such contexts, and their superior ability to use the north/south/east/west directional system is not utilised.

But much more fundamental differences can be found by examining the syntax and grammar of languages. An early work which explicitly addressed the mathematics in the syntax of a language was Gay & Cole's (1967) work concerning Kpelle mathematical concepts. Empirical evidence is presented for the conclusions that Kpelle find disjunction and negation considerably easier than American English speakers, and, further, "[the Kpelle] find disjunction easiest; in order of increasing difficulty are conjunction, negation, and implication. Equivalence they find very difficult. This pattern contrasts significantly with American behaviour, and many of the differences seem to reflect differences in linguistic structure between Kpelle and English" (Gay & Cole, 1967, p. 83).

Gay and Cole note differences in the way the Kpelle discuss and argue. However, they analyse the logic of the language in classical terms. Perhaps the differences in logical understanding reflect a more fundamental difference in the way relationships are expressed?

Many indigenous languages are being seriously affected by the dominant world language in the region: Maori in New Zealand by English, Yup'ik in Alaska by Russian and English, Euskara of Basque country by Spanish and French, and so on. As a result some of the grammatical constructions which indicate other ways of conceptualising quantity, relationships or space are being lost. For example, in modern Maori, number words are treated as they are in English: ‘three bottles’ has the number acting like an adjective in the same way as ‘red bottles’ or ‘tall bottles’. However recent work with older speakers of Maori has produced evidence of a different role for number words in traditional speech (Trinick, 1999). It was first noticed that number words in Maori are often used with verbal time markers. Many Maori sentences start with *e, kia, ka, kua* or *i*. These indicate tense. That the first three of these are often used with the number words indicates a verbal origin. Further evidence comes from the grammar of negation. Consider the three sentences:

E wha nga kina	=	There are four sea-eggs (Four the sea-eggs)
Kei te haere tatou ki Te Kaha	=	We are going to Te Kaha (Going we to Te Kaha)
He pouaka nui tenei	=	This is a big box (Box big this)

And now look at how each sentence is negated:

Kaore e wha nga kina, (e toru ke)	=	There are not four sea-eggs (there are three)
Kaore tatou i te haere ki Te Kaha	=	We are not going to Te Kaha
Ehara tenei i te pouaka nui	=	This is not a big box

The form of the sentence negating number is the same as the form of the sentence negating the action, and different from the sentence negating an adjective.

Thus, numbers were expressed as actions. In English this would be like saying “the bottles are three-ing on the table”. A similar use of number is found in Haida, a language spoken amongst First Nations people on the coast in north-west British Columbia. Here the verbal form is explicit: “Dii daghalang stingaagang = My brothers two. This is a sentence, in which ‘two’ [‘sting’ in Haida] is the verb. In English, of course, we would say "I have two brothers”. In Haida, one cannot ‘have’ brothers; brothers ‘are’. They exist –and being discreet and countable entities, they exist numerically” (Bringhurst, 1999).

Denny (1986) notes a verbal use of number words for Ojibway:

nis-iwag	=	they (animate) are three
nis-inoon	=	they (inanimate) are three
nis-ing	=	multiply by three (three times)

In mathematical talk we use numbers as objects, i.e. as nouns. Denny (1986) also reports numbers having a noun morphology in the Inuktitut language of Aivilingmiut. This is indicated by the use of noun suffixes:

one	atausiq	(none)	(singular noun)
two	marruuk	-uk	dual noun ending
three	pingasut	-t	plural noun ending

Thus, *pingasut*means *a group of three*. In order to say *three caribou*, you would say *pingasut tuktuit*, i.e. *a three-group of caribou* or *a caribou group-of-three*.

These alternative ways of talking about quantity do not constitute a mathematics. However, it is interesting to speculate on what sort of mathematics might have arisen had an extensive formalisation of quantity taken place in a linguistic environment where numbers were actions not objects. An experienced mathematician has noted that scalar quantity can be regarded as the first in a sequence of operators in analysis: function value, first derivative, second derivative, and so on (Butcher, 1998). In such a conceptualisation it might be more natural to think of scalar quantity as an action. Alternatively, what concept of the continuum might we have if we spoke of quantity as 'becoming one, becoming two, etc'?

Another aspect of the conventional way of making sense of quantity is the number line. Rulers are obvious examples, but also the base ten system is often represented to small children as bundles of marks (sticks, tallies) lined up together. Lipka (1994), however, gives an example where the number words of the Yup'ik seem to suggest a cyclic image rather than a linear one. Derived from body counting of fingers and toes, Yup'ik teachers have used a cycle based on twenty which is used to represent numbers symbolically –a representation approved by Yup'ik elders as in tune with their understanding.

A further example of linguistic difference in the idea of number is illustrated in the Indonesian language of Kedang. In that language the words *udeq, sue, tèlu, apaq, leme* are one, two three, four and five respectively. However, there are also the nouns *munaq* (one unit), *suen* (two units), ..., *lemen* (five units), etc (Barnes, 1982). Hence, multiplication is expressed abstractly as *lemen sue* (two lots of five units), which is different from *suen leme* (five lots of two units). In English, multiplication can be expressed as, say: 'five times two' or 'two times five' where the 'five' and 'two' can be interchanged without altering the word forms, the grammar, or the sense. In other words, commutativity is part of the language of multiplication in English, non-commutativity is the privileged form in Kedang. (Note that this is not to say that Kédang speakers cannot understand or express commutativity if they wish to do so).

The point being made is that, within each language, there are particular ways of expressing ideas of quantity, relationships or space. While it is possible to describe all these linguistic expressions in each language, there remains a question mark over whether the full complexity of the expression can be rendered in another language. So we are concerned not just with individual features of, say, whether a number is an action or a description or a thing in itself, but with the whole way in which quantity is approached in its myriad of instantiations and its relationship to, say, measurement, comparisons, or time. Then there is the further question as to whether such culture-specific concepts are formalised, or, if they were, what the result would be.

Focus on Ethos

Such questions cannot be answered simply by examining the lexicon (conceptual repertoire), syntax or grammar of a particular language. Rather we need to focus on the interaction between these linguistic characteristics and the overall metaphysics expressed by what we might call the ethos, or world-view, of a given speech community. In order to illustrate this idea, we will turn to another indigenous language which, although embedded in an Indo-European linguistic environment, is very different in its ethos. Euskara is the language of the Basque people of northern Spain / southern France. It is an important example because virtually all of the speakers of Euskara are bilingual, either in French or Spanish. Consequently, their schooling is done through recourse to Indo-European language models of number construction even though the language of schooling is often Euskara. These Indo-European cognitive models are taking over and affecting the indigenous structures in profound ways. The two models struggling with each other inside one linguistic system, provides us with an interesting vantage point from which two radically different systems can be examined.

In Euskara (and in certain other non-western languages, e.g. Yucatec Maya) the ontology of 'being' is differently positioned. In describing it we are hampered by the spatiotemporal particulars (Watson, 1990) imposed by English, which enculturate us to perceive "a universe consisting of (a) void or 'holes', and (b) substance or matter which has 'properties' and forms island-like 'bodies', [and] an absolute unbridgeable difference between the matter and the 'holes'..." (Whorf, 1938). Thus, in English the 'matter' is set against a backdrop of 'nothingness'; i.e. the 'void' sets off the object itself. The 'void' is passive in terms of its meaning-making.

However, in Euskara what is the 'void' (from the point of view of English) is completely 'full'. Thus, 'matter' becomes the 'ground' and forms are cut-out from it, as cookies from undifferentiated dough. Thus, what's inside the figure and what is outside of it is the same 'stuff'. This cutout operation is accomplished linguistically by manipulating suffixes indicating particular qualities, shapes and modes of being and extension. In this sense it could be argued that the ontology intrinsic to Euskara concentrates far more attention on constructing the boundaries, borders or edges of the 'matter-stuff', i.e. on shaping it. It could be argued that the ontology intrinsic to Euskara is one that concentrates its attention far more on what would be understood, in English, as 'negative space' or, perhaps, as the space between the outlines of objects. Hence the need for a mind-set that recognises these spaces as real –as real as the 'positive' forms of English.

It is interesting to compare this with Watson's description of Yoruba, an African language:

> The objects which Yoruba speakers are committed to saying there are in the world are sortal particulars - material objects defined through their particular nature. Certain sets of characteristics form *definitive boundaries* of the material objects that Yoruba speakers talk about as being infinitely scattered through space and time. [emphasis added] (Watson, 1990, 297)

Some of the flavour of this way of viewing the world can be understood with respect to an atomic scientist's view of, say, a table. The table is understood to be made up from atoms and molecules which are not different from those in the air around it, the floor on which it stands, or the person viewing it. Indeed, on our present understanding of atoms, there is much more 'empty space' within the table than there is 'matter'. The table, therefore, is simply a particular collection of molecules –what we observe is these molecules 'table-ing'.

But even this different way of conceiving is still affected by our English perception and its emphasis on spatial form. In Euskara the form of an object is also defined by the 'mode' of its being, that is, by its qualities. Thus, physical shape and whether it exists now or existed in the past are not as relevant as they are in English. A way to illustrate this point is to explain, for example, that an Euskara-English Dictionary would give *su* for *fire*. But *su* corresponds to the inner intrinsic nature of fire. If we wish to talk about this fire-stuff as an entity extended in time and space as an event, as in the English "There was a fire on Oak Street", then the suffix *-te* must be added to indicate this: *sute*. As another example, in English the sentence "he saw four dogs" speaks of four objects in space with the size, shape and characteristics of a dog that were observed; in contrast, in Euskara "Lau txakur ikusi zuen" speaks of matter with the characteristics of dogness and fourness that was seen. Note that in this example number is realised as a possible mode of being, (cf. the Haida examples above and similar ones in Yoruba discussed by Watson, 1990).

Levinson refers to similar linguistic implications in his discussion of Lucy's (1992a, p. 73ff) work on Yucatec (Levinson, 1996, p. 185):

> Like Tzeltal, Yucatec has a developed set of numeral classifiers. The motivation, Lucy claims, is that nominals in Yucatec fail, by themselves, to individuate entities. It is only by collocation with a numeral classifier or some other shape-discriminating phrase that such nouns can come to designate countable entities. This thesis, carried to its logical extreme, would amount to the claim that all nominals in Yucatec are essentially 'mass' nouns and that the language makes no ontological commitment to 'entities' as opposed to materials, essence or 'stuff' at all. In order to individuate entities, a numeral classifier or some predicate is required to impose individuation on the material, metaphorically in much the same way that a cookie-cutter cuts up undifferentiated dough!

The idea that objects only come into being when the word for their essence has some kind of classifier attached to it would help to explain the Tzeltal insistence on specifying the geometrical nature of the figure. Thus, it is not only numbers that are conceived as qualities, but also geometric figures. Other works dealing with geometric conceptualisations are Pinxten, van Dooren & Harvey (1983) and Pinxten, van Dooren & Soberon (1987).

The suggestion contained in these paragraphs that indigenous languages such as Euskara, Yoruba, Yucatec Maya and Haida embody a metaphysical view of the world with common elements is probably a reflection of the lack of understanding we have of the subtle differences between the world-view of each of these languages. To us as English speakers they are so different from our understanding that they just seem the same. From a typological point of view, it is unlikely that these linguistic systems, drawn from four continents, would appear so similar if additional aspects of them were subjected to analysis.

Sapir and Whorf, American linguists of the first half of this century, wrote explicitly of the way languages codify a particular dissection of nature. To quote Lee (1996, p.113):

> Thus both Sapir and Whorf made it clear that the possibility of comparing geometries is based on an implicit assumption that what they systematize in the first instance is a common reality –space as it may be experienced by human beings– and that this is differently described and mentally organized according to principles embodied in each geometrical method. [...] It may be useful to give a little more attention to the way different geometries (which are effectively different mathematical perspectives brought about by different ways of talking) articulate different conceptions of space.

Most writers who have attempted to examine spatial questions in non-western languages, have started from the unexamined premise that the objects themselves are in need of no further elaboration and can be equated safely with those found in English. As Levinson (1996, p. 191) has noted, Lyons (1977:438ff) as well as other writers have pointed that we only identify nouns, verbs, and so on in another language on the basis of a mix of syntactic and ontological criteria. Consider again Lucy's conjecture about Yucatec 'nouns' as denoting material or essence, not objects as is the case in English: that would make the ontological prototypes for 'nouns' in Yucatec be a property and not an entity. We need to keep in mind that the conceptual frames initially brought into play in linguistics (and mathematics) were those readily available to Standard Average European speakers. In summary, it is not just the way in which the universe is understood, but also the way in which we talk about language, the very terminology we use, which may not be appropriate when we are discussing linguistic systems fundamentally different from English and other Indo-European languages.

There is the additional problem that on occasion the referential object produced in, say, Euskara or Yoruba, appears to coincide with that of a spatiotemporal particular in English. That apparent translatability is particularly deceptive because such a case makes the language learner or linguist assume that this similarity can be extended to the system as a whole (Watson, 1990). Furthermore, the impact of the ontology of a dominant SAE language on that of minority one like Euskara or Yoruba can result in the introduction of subtle cognitive shifts that, in turn, encourage additional copying of the QRS system of the prestige language into the minority one.

Implications for Mathematics Education

What does all this mean for mathematics education? First of all it should be said clearly that these ideas do not mean that different peoples are limited by their language to the concepts expressed in that language or to the ethos it embodies. This chapter is an example of the way that it is possible to consider ideas which have arisen in other language structures. Thus, the mathematical, or QRS, ideas that might emerge from a study of an indigenous (or any) language, add to the potential concepts from which formal mathematics may draw, or act as a creative source for speakers of other languages. This applies as easily, for example, to ideas moving from English to Haida as it does for those moving from Haida to English.

The next point is that the use of culturally specific resources to achieve the conventional aims of mathematics education is still a very open question. One the one hand, the existence of fundamentally different networks of image-schemata (sometimes referred to as 'world-views'), including those aspects regarded as mathematical, calls into serious question the use of isolated materials from other cultures in the promotion of educational objectives from, say, an English or European culture. For example, the use of weaving patterns or number words from other cultures in the service of geometry or numeracy is likely to devalue those materials by stripping them of the linguistic and practical contexts in which they are meaningful. There may be other educational reasons for their use, (e.g. motivation or creating links with particular students), but such practices are likely to need constant re-evaluation of their effectiveness and an awareness of hidden consequences. For example, if practices which can be evaluated as elementary in conventional mathematics terms are presented as the ‘mathematics’ of a particular culture, then there is a danger of that culture being labelled as ‘primitive’. On the other hand, without presenting ideas from fundamentally different image-schemata, it is not possible to illustrate the way in which conventional mathematics has developed in a particular way which could have been different. Bishop promotes this idea with his concept of using cultural conflict within the enculturation of mathematics (Bishop, 1994).

A further implication for classroom mathematics is that the evidence presented above indicates a fundamental relationship between mathematical thinking and language –and this means any language, including the specialised language of a mathematics classroom. There is increasing research into how mathematical discussion affects the concepts which are formed. However, there is a need to open this up to consider the image schemas embedded in the ontology of the language, not just the differences between, say, informal and formal language. For example, do new learners of mathematics have some fundamental concepts which get suppressed in the environment of a conventional mathematics classroom? Will increased classroom talk allow unconventional schemas to be valued, and will this increase mathematical ability? Will increased talk just increase the distance (measured in academic achievement) between those who quickly adopt conventional schemata and those who do so only slowly?

As far as teaching is concerned there is a challenge to educate mathematics teachers, particularly monolingual ones, to review their conceptions of mathematics as a more contingent subject than that which was taught to them. It will make it more difficult to present the content of mathematics as “how the world is”: teachers will need to develop ways of talking about their subject which bring out the conventional nature of its concepts, and not just its symbolism and methods.

The main implication for mathematics education, however, concerns learners from indigenous or minority cultures, who are often disadvantaged with respect to mathematical achievement in conventional terms. It has long been known (e.g. D’Ambrosio, 1990) that the basis for this lies in the cultural estrangement of studying a field of knowledge which has been developed through another world-view. D’Ambrosio refers to the way this manifests in the classroom and the community as the ‘social terrorism’ of mathematics. Overcoming this estrangement is no easy task, but acknowledging the problem must be an essential feature.

Such acknowledgment must come not just in the minds of the educators, but also in the curriculum as it is received by students. One attempt at this is described by Lipka (1994, p. 25):

> The pressure behind developing a Yup’ik mathematics is three-fold:
> 1) to show students that mathematics is socially constructed;
> 2) to engage students in a process of constructing a system of mathematics based on their cultural knowledge;

3) to connect students' knowledge of 'their mathematics' through comparisons and bridges to other aboriginal and Western systems

In other words, access to the conventional, widespread field known as 'mathematics' must come through the world-view in which it has been developed and is mostly expressed: that of Indo-European languages. If your network of image-schemata is different from the dominant one, then the first step is to understand the role of your own world-view in making sense of quantity, relationships and space, so you can appreciate another one. It should be noted, in addition, that a facility with non-Indo-European ontologies and QRS systems may shed a new light on concepts that exist in the dominant culture of mathematics.

Such an educational task seems to place an added burden on anyone who is starting from a different world-view than that of the knowledge they are seeking. In a sense this is true, but there are two important points to be made. There is evidence (Cummins, 1986) that bilingual learners, provided they have a 'threshold fluency' in both languages, have a cognitive advantage in any educational task. Knowing a language implies an intuitive understanding of a whole network of image-schemata. Perhaps knowing a language well enough to internalise its world-view is the criterion of the 'threshold fluency' below which the cognitive advantage does not occur? Thus, cognitive advantage might be interpreted to mean that the sort of knowing which results from having two or more world-views is a deeper, more aware sort of knowing than that which results from having only one. If this is right, then it is probable that the more dissimilar are the representations of reality which the learners have access to, then the greater will be their potential for perceiving differences between the two linguistic systems and gaining from that experience. Hence, the added burden mentioned above does not mean that people from a different world-view have to do more to reach the same place, rather that they are going to a different, deeper place.

The second point is that, if someone already inhabiting the world-view of, say, conventional mathematical knowledge wishes to reach this deeper level of understanding, then they also have an added task. It is a feature of many education systems, especially monolingual English-speaking ones, that such an alternate kind of understanding is not even recognised. To quote Whorf (1956):

> ... but to restrict thinking to the patterns merely of English ... is to lose a power of thought which, once lost, can never be regained. ... I believe that those who envision a future world speaking only one tongue ... hold a misguided ideal and would do the evolution of the human mind the greatest disservice. Western culture has made, through language, a provisional analysis of reality and, without correctives, holds resolutely to that analysis as final. The only correctives lie in all those other tongues which by aeons of independent evolution have arrived at different, but equally logical, provisional analyses (p. 244).

Conclusion

There is an exciting, unexplored challenge for the linguistic suggestions mentioned in this paper to be further researched. Specific questions for mathematics educators include the following.

- Are there children for whom the (conventionally) 'basic' mathematical concepts (e.g. number, shape, sets, symmetry, logical relations) are not readily available because of conflicting (or incommensurable) concepts powerfully present in their own cultural-linguistic heritage?

- If (as the writers believe) such children exist, how can their mathematical thinking best be acknowledged, and can it be maintained at the same time as learning conventional mathematical ways of thinking?
- What are the conditions under which such children have a cognitive advantage in mathematics, and what is the nature of this advantage?
- What is the contribution of teaching mathematics as a universal subject with respect to the phenomenon variously known as math-phobia or the social terrorism of mathematics?
- If (as the writers believe) it is substantial, what needs to happen in teacher education, in classroom practice, and in social attitudes to the subject, in order that significant changes occur?

Another question, which is outside the scope of this paper, but which is critical if change is to take place, is whether sufficiently strong evidence can be provided to convince mathematicians that their subject can be seen as relative at a fundamental level. Searching for this material within mathematics itself, and presenting it appropriately is an important next task.

All the above ideas have been approached from different directions by ethnomathematicians, anthropological and cognitive linguists, educationalists and cultural psychologists. The ideas are particularly important because of the fundamental nature of conceptions of quantity, relationships and space. To quote Whorf (1956) once more, and this time with more insight into what he meant:

> ... an important field for the working out of new order systems, akin to, yet not identical with, present mathematics, lies in more penetrating investigation than has yet been made of languages remote in type from our own (p. 255).

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